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Catenary action in rebars crossing a casting joint loaded in shear

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Abstract

Reinforcement crossing a casting joint loaded in shear exhibits catenary action as the shear displacement increases. The load carrying capacity of such a joint is in practice often calculated by use of empirical methods to account for shear friction effects or by a first order plastic analysis if dowel action is included. The strength increase/reserve due to catenary action in the rebars is often neglected; however in some cases it may be necessary to utilize the effect in order to ensure overall structural robustness. This paper presents results of a study, where the increased shear capacity due to catenary action was investigated experimentally in a simple push-off setup and theoretically by a second order plastic analysis. The model captures the combination of dowel and catenary action with increasing shear displacement and satisfactory correlation between the S-shaped test results and theory is found when reasonable material properties are assumed.

1 Introduction

Dowel action in reinforcing bars embedded in concrete is a well-known phenomenon which can be utilized as load carrying mechanism in structural concrete. Inclusion of dowel action in design is primarily relevant for problems that involve transfer of shear through casting joints or connections such as on-site assembly of precast concrete structures, see e.g. *fib* Bulletin 43 (fib 2008). Fig. 1 schematically illustrates a model for shear transfer through a (perfectly smooth) concrete interface by activation of dowel and catenary action in a rebar.

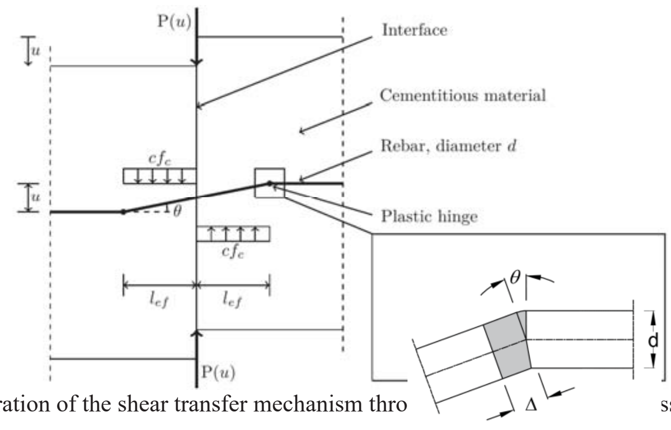


Fig. 1 Illustration of the shear transfer mechanism thro ssed by a rebar

The case of pure dowel action is most clearly observed when a rebar is partly embedded in a large block of concrete while being loaded by a transverse force at the free end. In most practical cases, the load carrying capacity will be governed by development of a plastic hinge in the rebar. The position of the plastic hinge depends on many parameters. However, in situations with practical relevance, the hinge develop somewhere in the embedded part of the rebar. In this case, local crushing of the concrete will take place when the hinge develops. As the concrete is subjected to a local contact pressure beneath the rebar (i.e. partially loaded), a tri-axial stress state can develop which increases the resistance of the concrete beyond its uniaxial compressive strength. The first attempt to study this basic problem was carried out by Friberg (1938) who modelled the dowel as a linear elastic beam supported on an elastic foundation. However, since both materials behave plastically at the ultimate state, a model based on the theory of plasticity seems more appropriate. Such a model was developed by Højlund Rasmussen (1963), who used test results to empirically determine the strength enhancement factor for the concrete (i.e. the strength of the partial loaded area). Rasmussen's model included eccentricities in the shear loading. The strength enhancement factor depended among other factors on the degree of confinement from the surrounding concrete as well as the amount of surrounding reinforcement.

In contrast to the basic problem studied by Højlund Rasmussen (1963), Fig. 1 illustrates a so-called two-sided dowel joint, where the rebar is fully embedded in concrete on both sides of the casting joint. In this case, pure dowel action is only the first phase of the load transfer mechanism of the joint. Dowel action will gradually be accompanied, and eventually replaced, by catenary action in the rebar when the shear displacement, u, increases. Development of full catenary action requires relatively large shear displacement and for this reason, this effect is seldom utilized in the ultimate design of structural joints. However, for assessment of structural robustness as well as verification of structural performance under accidental load cases, catenary action can play an important role.

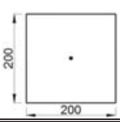
Most of the studies carried out regarding a two-sided dowel joint have been performed in the context of shear or punching resistance of concrete beams and slabs. Dulácska (1972) made tests on rebars crossing a crack, where the rebar was inclined at an angle with respect to the crack. It was reported that the behavior of the specimens was almost ideally elastoplastic and that the relationship between the shear and tension forces that developed in the rebar was non-linear.

In the case of joints in precast structures, the resistance of bolted connections joining two precast elements loaded in pure shear was investigated by Engström (1990), who suggested a model for the load carrying capacity based on a combination of dowel action and friction. The model determined a capacity, which depended on the axial restraint of the dowel. The test results showed an initial elastic behavior followed by a pronounced plastic behavior, which is in agreement with the general behavior of a two-sided dowel joint. In this paper, we present the results of a preliminary experimental program, which aims to study the effect of catenary action on the shear behavior of frictionless joints crossed by a rebar. In addition, the paper presents a simple model for the load-displacement response of the test specimens. The model has been formulated on the basis of the upper bound theorem of rigid-plasticity and accounts for large displacements through non-linear relationships between displacement and internal plastic strains.

2 Experimental program

The experimental program consisted of three preliminary specimens cast with mortar, Series P, and six specimens made of regular concrete with aggregate sizes up to 16 mm, Series A and B. The series were designed to investigate the enhancement factor of the two materials. All specimens were provided with one single centrally positioned rebar, see Table 1, with diameter d = 8 mm crossing the casting joint at an angle of 90°. Each specimen was cast in two sequences. In the first sequence, half of the specimen was cast in a smooth mold. Subsequently, the joint surface was treated with grease to minimize friction in the interface before casting the remaining part of the specimen. The area of the joint interface was 200x200 mm and the overall dimensions of the specimens were 200x200x400 mm, see Table 1 for specifications of the material properties.

Table 1 Overview of specimens in experimental program

	Material	Position	f_c [MPa]	f_y [MPa]	f_u [MPa]	$\Delta_u [mm]$	Replicates
P	Mortar/Mortar		27.8/29.9				3
A	Concrete/Concrete		42.8/44.7	614	729	2.5-3.5	3
B	Concrete/Concrete		45.7/49.3				3

2.1 Test setup

The specimens were tested in a classical push-off setup, where the thrust line of the applied loads coincided with the plane of the casting joint in order to simulate pure shear loading; see Fig. 2 (left). In Fig. 2 (right) a deformed specimen during testing can be seen.

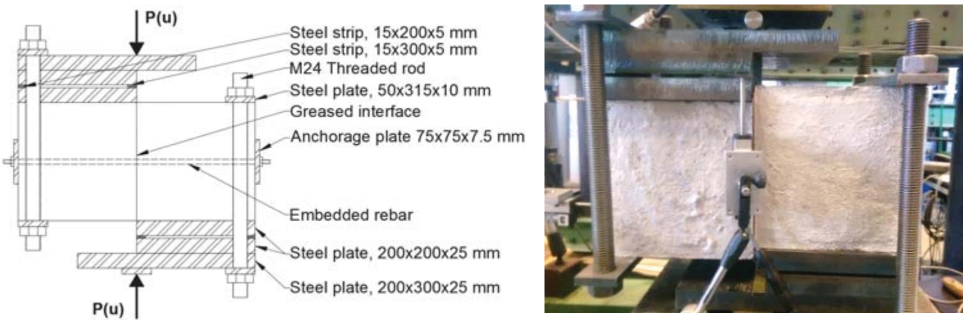


Fig. 2 Push-off test setup for shear loading (left) and a specimen during test (right)

In the setup, the flow of forces was ensured by thin steel strips placed at the outermost positions of the reaction plates. In addition, threaded rods were used to clamp the reaction steel plates to the specimen. The centre strip was also used to measure the relative shear displacement on each side of the specimen and the reported test results are given as an average of the two measurements. The tests were conducted in static displacement control with a constant displacement rate of 2.5 mm/min.

The properties of the reinforcement were tested in uniaxial tension and the strain development was recorded using a single 36.3 megapixel digital camera for 2D digital image correlation in the Aramis software (GOM 2009). The behavior was recorded over the total length of approx. 300 mm between the grips of the hydraulic testing machine. The surface was spray painted with a white basis color followed by random sprayed black dots to create a unique and recognizable pattern on the surface. The surface of a rebar is not plane; however the measurements were carefully conducted at the midsection of the rebar. As the test condition was uniaxial the deviation from the measured 2D case to the actual 3D case is regarded as insignificant. The concrete strength was determined from compression tests on ϕ100x200 mm cylinders.

2.2 Experimental results

The experimentally obtained load-displacement curves are shown in Fig. 3 (upper left). In general, an almost linear response is observed up to a load that approximately corresponds to the dowel-solution suggested by Højlund Rasmussen (1963). Beyond this level, the load was able to increase with further displacement. The response curve in this regime first followed a concave development up to a displacement of approximate 8-9 mm followed by a convex development until the reinforcement ruptured at a shear displacement of approximate 16-20 mm. The ultimate shear displacement capacity was for all concrete specimens of governed by tensile rupture of the rebar, see Figure 3 (bottom left).

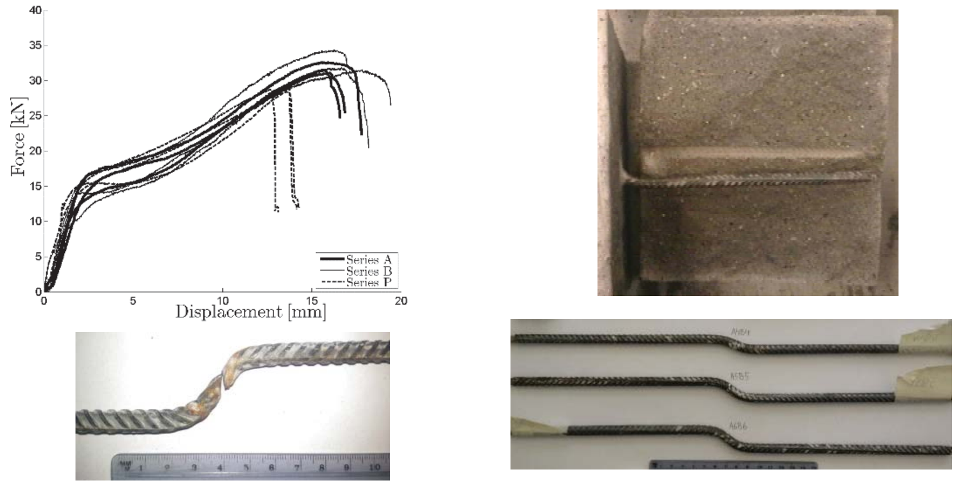


Fig. 3 Load-displacement relations (upper left), rupture of rebar (lower left), premature splitting failure of Series P (upper right) and deformed rebars of series P, (lower right)

For specimens made of mortar, Series P, the load-displacement responses were similar to the curves obtained from tests on concrete specimens, see Figure 3 (upper left), however the tests were terminated by a premature splitting failure of the mortar, see Figure 3 (upper right). Fig. 3 (lower right) also shows the deformed shape of the rebars embedded in specimens suffering splitting failure. The pictures indicate that plastic hinges had formed on each side of the casting joint with no additional plastic deformation in the remaining part of the rebar.

3 Second order plasticity model

A model for the inelastic regime of the load-displacement responses of (frictionless) two-sided dowel joints is presented in the following. It is assumed that the concrete and the reinforcing steel are rigid-perfect plastic materials obeying the associated flow rule. In the model, displacements are therefore the sole results of accumulated plastic deformations. The problem is treated as a static displacement controlled problem, where the shear displacement, *u*, is considered as a monotonic function of time. For convenience, a displacement velocity equal to unity is assumed:

$$u(t) = t \tag{1}$$

The starting point of the model formulation is to assume a deformation shape for the rebar. As *u* increases, the zones with plastic hinges undergo plastic rotation, θ, as well as elongation, Δ, see Fig. 1. The rates of the plastic deformations can be determined by establishing the kinematic relationship for the problem. Then, by assuming the normality condition of plastic theory and by applying the work equation for increments of displacement, the necessary equations to relate the applied shear load *P* to the displacement, *u*, can be derived. This form of upper bound plastic analysis where large displacements are considered has also been carried out by e.g. Calladine (1968), Bræstrup (1980), and Belenkiy (2007).

3.1 Material properties

For local tri-axial stress conditions of the type developed in the concrete beneath the dowel, an effective compressive strength, f_{cc}, is assumed:

$$f_{cc} = cf_c \tag{2}$$

Here *c* is the enhancement factor for the concrete compressive strength which has to be determined by calibration with test results. Højlund Rasmussen (1963) found *c*-values in the range of 3.7-5.4 from tests on one sided dowels. However, it should be noted, that the rebars investigated by Rasmussen had relatively low yield strength compared to modern reinforcing steel. In addition, the stress condition in the concrete beneath a one-side dowel is not directly comparable with that of a two-sided dowel.

The characteristics of the reinforcement are determined by tension tests. The local deformation capacity of the rebars is of particular interest for the model. In this study it has been determined by measuring the deformation over a characteristic length of approximately 2*d*, covering the area of the highest local plastic strains, see Fig. 4 (right). The results for the reinforcement used in this study can be seen in Fig. 4 (left). As input for the model, f_y = 614 MPa and f_u = 729 MPa are used as the lower and upper limit for the steel strength, while the interval for the maximum elongation in a plastic hinge is taken as Δ_u = 2.5-3.5 mm.

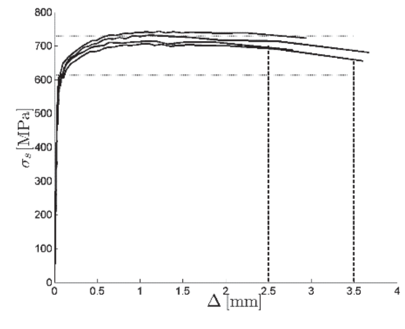


Fig. 4 Measured σ_s-Δ relationship over a characteristic length of 2*d* for a reinforcing bar (left) and picture of strain localization (necking) in a rebar just before rupture (right)

3.2 Kinematic relations of the plastic hinge

The length, l_{ef}, defines the position of the plastic hinges, see Fig. 1. The length is estimated by the one-sided dowel solution established by Højlund Rasmussen (1963):

$$l_{ef} = \frac{d}{\sqrt{3}}\sqrt{\frac{f_y}{f_{cc}}} \tag{3}$$

From the predefined shape of deformation (see Fig. 1), the following relationship can be established between shear displacement, u, and angle of rotation, θ, in the plastic hinges:

$$\tan(\theta) = \frac{u}{2l_{ef}} \tag{4}$$

To accommodate the change of geometry when u increases, it is necessary to impose elongation in the rebar in addition to the development of plastic hinges. Since rigid-plastic material behavior has been assumed, it is allowed and convenient to consider the elongation as a plastic extension, Δ, concentrated in the plastic hinges (as indicated in Fig. 1). In this way, θ and Δ may be regarded as the general strains in the plastic hinges, which are subjected to general stresses in the form of bending moments M and normal forces N. The following relationship between u and Δ applies:

$$l_{ef}^2 + \left(\frac{1}{2}u\right)^2 = (l_{ef}+\Delta)^2 \Rightarrow \Delta = -l_{ef} + l_{ef}\sqrt{1+\left(\frac{u}{2l_{ef}}\right)^2} \tag{5}$$

By utilizing Eqs. (1), (4) and (5), the rates (i.e. velocities) of plastic deformations in the hinges can be expressed as follows:

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{d\theta}{du}\frac{du}{dt} = \frac{d\left(\arctan\left(\frac{u}{2l_{ef}}\right)\right)}{du}\cdot 1 = \frac{1}{2l_{ef}}\frac{1}{1+\left(\frac{u}{2l_{ef}}\right)^2} \tag{6}$$

$$\dot{\Delta} = \frac{d\Delta}{dt} = \frac{d\Delta}{du}\frac{du}{dt} = \frac{d\left(-l_{ef} + l_{ef}\sqrt{1+\left(\frac{u}{2l_{ef}}\right)^2}\right)}{du}\cdot 1 = \frac{u}{4l_{ef}}\frac{1}{\sqrt{1+\left(\frac{u}{2l_{ef}}\right)^2}} \tag{7}$$

which leads to the following condensed expression for the kinematic condition of the problem:

$$\frac{\dot{\Delta}}{\dot{\theta}} = \frac{u}{2}\sqrt{1+\left(\frac{u}{2l_{ef}}\right)^2} \tag{8}$$

3.3 Constitutive relationship for the rebar

The onset of plastic deformation in the rebar in the case of pure tension, Δ, is possible when the cross section is subjected to the plastic tensile capacity N_p. In the case of pure bending, plastic deformation, θ, is possible when the cross section is subjected to the plastic moment capacity M_p. These sectional forces are determined as:

$$N_p = \frac{\pi}{4}d^2 f_y \tag{9}$$

$$M_p = \frac{1}{6}d^3 f_y \tag{10}$$

where d is the cross sectional diameter and f_y is the yield stress of the rebar. For combined loadings, i.e. when the cross section is subjected to (M, N), plastic deformations may initiate when the yield condition of the cross section is equal to zero, $f(M,N) = 0$. For circular cross sections, the exact yield condition contains trigonometric relations, which are difficult to handle in a closed form analytical manner. Therefore, for simplicity, the analytical non-linear yield condition for a rigid-plastic rectangular cross section is adopted in the following:

$$f(M,N) = \frac{M}{M_p} + \frac{N^2}{N_p^2} - 1 = 0 \tag{11}$$

It can be shown that (11) is a reasonable and conservative approximation to the true yield condition when Eqs. (9) and (10) are used to determine N_p and M_p, respectively. The same approximation has previously been used for similar problems (Millard and Johnson 1984). Now, according to the assumed normality condition (i.e. the associated flow rule), the rates of plastic deformations must fulfil the following constitutive relationships:

$$\dot{\Delta} = \lambda \frac{\partial f}{\partial N} = 2\lambda \frac{N}{N_p^2} \tag{12}$$

$$\dot{\theta} = \lambda \frac{\partial f}{\partial M} = \lambda \frac{1}{M_p} \tag{13}$$

where λ is a positive constant proportional to the displacement velocity. The constant vanishes when Eqs. (12) and (13) are used to determine the ratio of plastic strain rates:

$$\frac{\dot{\Delta}}{\dot{\theta}} = \frac{4}{3\pi} d \frac{N}{N_p} \tag{14}$$

3.4 Determination of sectional forces in plastic hinges

The set of sectional forces ($N(u)$, $M(u)$) that develops in the plastic hinges for any given value of shear displacement, u, can now be determined by equating the right hand sides of Eqs. (8) and (14) and afterward imposing $f(N,M) = 0$ according to (11). The results are:

$$\frac{N(u)}{N_p} = \frac{3\pi}{8}\frac{u}{d}\sqrt{1+\left(\frac{u}{2l_{ef}}\right)^2} \leq 1 \tag{15}$$

$$\frac{M(u)}{M_p} = 1 - \frac{9\pi^2}{64}\frac{u^2}{d^2}\left(1+\left(\frac{u}{2l_{ef}}\right)^2\right) \geq 0 \tag{16}$$

3.5 Load displacement response

Based on the relations between u and the general stresses in the plastic hinges, i.e. Eqs. (15) and (16), the load displacement response of the joint can be established by use of the work equation. The external work for an increment of displacement, δu, is found as:

$$\delta W_E = P(u) \cdot \delta u \tag{17}$$

The internal work has contributions from the energy dissipated in the plastic hinges as well as the energy absorbed when the concrete crushes beneath the rebar. The following formula can be derived from the mechanism shown in Fig. 1:

$$\delta W_I = 2 f_{cc} d l_{ef}\left(\frac{1}{4}\delta u\right) + 2M(u)\delta\theta + 2N(u)\delta\Delta \tag{18}$$

According to Eq. (1), it is given that $\delta t = \delta u$ and thereby the incremental rotation and elongation of the plastic hinges are related to the shear displacement increment in the following way:

$$\delta\theta = \dot{\theta}\delta u \qquad \text{and} \qquad \delta\Delta = \dot{\Delta}\delta u \tag{19}$$

Now, by inserting (3), (15), (16) and (19) into (18) and by setting up the work equation, we arrive at:

$$P(u) = \begin{cases} \dfrac{d^2}{2\sqrt{3}}\sqrt{f_y f_{cc}} + \dfrac{d^2\sqrt{f_y f_{cc}}}{2\sqrt{3} + \dfrac{3^{3/2} u^2 f_{cc}}{2d^2 f_y}} + \dfrac{3^{3/2}\pi^2 u^2}{128}\sqrt{f_y f_{cc}} & \text{for } N \leq N_p \\ \dfrac{d^2}{2\sqrt{3}}\sqrt{f_y f_{cc}} + \dfrac{\sqrt{3} u \pi d \sqrt{f_y f_{cc}}}{\sqrt{64 + \dfrac{48 u^2 f_{cc}}{d^2 f_y}}} & \text{for } N = N_p \end{cases} \tag{20}$$

As the normal force reaches the plastic capacity, the hinges will lose their ability to carry bending moments and the response of the joint will depend solely on the tensile action in the rebar, i.e. catenary action. The capacity of the rebar is exhausted once the elongation of the rebar in a plastic hinge, Δ, reaches the maximum elongation, Δ_u, as defined in Section 3.1. The model is theoretically applicable for any dimension of rebar and any type of cementitious material when appropriate material properties are used and when other failure mechanisms are not governing. However, the model has so far only been experimentally validated for rebars of smaller diameter (8 mm) and normal strength concretes.

4 Comparison of tests with theory

Fig. 5 contains a comparison of the test results with the model presented. Only test results from Series A and B are included as the specimens from Series P experienced a premature mortar failure. The test results display some scatter which may be due to the inherent quasi-brittle properties of the concrete material and the influence of the casting procedure on the aggregate distribution, as the resulting capacity is highly affected by local conditions around the rebar. However, the results exhibit the same general S-shaped behavior and it is clear that the model captures this behavior in overall terms. It should be noted that the deviation increases with increasing shear displacement.

The model is shifted to start at an initial displacement of 2 mm (as indicated in the figure) where the load-displacement response indicates a shift from elastic to plastic deformations, i.e. the point where the plastic moment capacity of the rebar is reached. The black lines represent calculations performed with the minimum and maximum strength properties observed and the band in between represents the possible combinations. The yield strength of the reinforcement is varied from the observed yield strength, f_y, to the ultimate strength, f_u, to account for any hardening behavior of the rebar. The concrete strengths used are the maximum and minimum observed in Series A and B, see Table 1. The enhancement factor is assumed constant with a value of c=5.4, the maximum found by Højlund Rasmussen (1963). It is seen, that the c-factor is of the correct order of magnitude, as the initial plastic responses at small displacements are captured. In Fig. 5, the transition from a combination of moment and normal force to pure catenary action is indicated with a cross. It constitutes the point where the load-displacement behavior changes from concave to convex.

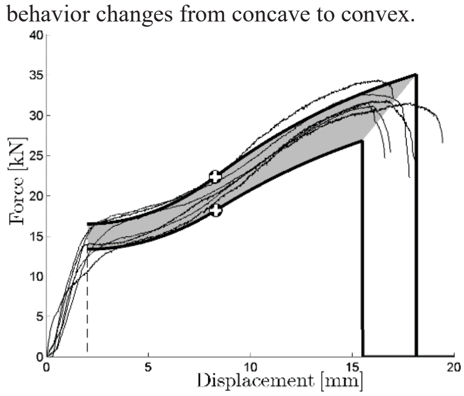


Fig. 5 Test results from series A and B compared to the second order plasticity model

5 Discussion

The model is simple, where the only calibration parameter is the enhancement factor. Refinements can e.g. be made by inclusion of friction depending on the interface properties. In the current tests, friction is ignored as the interface is greased, but in practice friction will be present. The magnitude is dependent on the clamping force (mobilised by tension in the rebar) and the friction coefficient. The friction contribution will mainly affect the load – displacement response at large displacements and thereby possibly reduce the deviation between the model and the test results.

In the present study, the enhancement factor of the concrete material is assumed constant; however it is debatable if the concrete material can maintain constant resistance as the displacement increases. Nevertheless, the estimation of the initial enhancement factor of the concrete will be of the correct order of magnitude independent of the choice of friction coefficient, development of the c-factor or steel stress-strain relationship.

In addition to the possible inclusion of the aforementioned parameters in the model, the prospects for further experimental work is e.g. to test more than one dowel crossing the casting joint, in order to investigate any group effect on the local enhancement factor for the concrete material. The influence of the aggregate size on the enhancement factor is also relevant for development of a model which accounts for different materials on each side of the casting joint, e.g. as in the case of on-site grouted narrow lapped joints in precast concrete structures.

6 Conclusion

The results of the experimental program presented in this study revealed an elasto-plastic behavior of a two-sided dowel joint. The load resisting mechanism was found to be governed by a combination of dowel and catenary action, resulting in an S-shaped load-displacement response in the plastic region. The test response was estimated by a second-order plasticity model, which captured the general behavior of the test results well and predicted the transition between a combination of dowel and catenary action to pure catenary action. The tested value of local plastic elongation in a rebar corresponded well with the test range, accurately predicting the termination of the test by reinforcement rupture.

The current layout of the model contains only one calibration parameter; the enhancement factor for the local concrete compression strength, c. Inclusion of additional parameters, such as friction contribution of the interface may provide better results.

Acknowledgement

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